EEL3701

Menu

- Boolean Algebra
  - > Theorems
  - > Definitions

See examples on web:
`BooleanAlgebra.PDF`

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The Mathematics of Logic Design - Boolean Algebra

- **Basic Postulates & Theorems**
  - > Identity Laws
    1. X + 0 = X X • 1 = X
    2. X + 1 = 1 X • 0 = 0
  - > Indempotent Laws
    3. X + X = X X • X = X
  - > Involution & Complementarity Laws
    4. (X')' = X
    5. X + X' = 1 X • X' = 0
  - > Commutative Laws
    6. X + Y = Y + X X • Y = Y • X
  - > Associative Laws
    7. (X+Y)+Z=X+(Y+Z)=X+Y+Z (XY)•Z=X•(YZ)=XYZ
  - > Absorption Laws
    8. X • (X +Y) = X X + (X•Y) = X

EEL3701 The Mathematics of Logic Design - Boolean Algebra

- **Important Theorems**
  - > Distributive Laws
    - 9. X•(Y+Z) = X•Y+X•Z X+(Y•Z)=(X+Y)•(X+Z)
  - > De Morgan's Law
    - 10. (X+Y+Z)' = X'•Y'•Z' (X•Y•Z)' = X'+Y' +Z'
  - > Duality
    - If we treat **+ and •** as dual pairs, and **0 and 1** as dual pairs, the theorems in one column can be deduced from the other, e.g.,

```
1.  X + 0 = X     2.  X + 1 = 1     5.  X + X' = 1
    ↓   ↓   ↓   ↓       ↓   ↓   ↓   ↓       ↓   ↓   ↓    ↓
1D. X • 1 = X     2D. X • 0 = 0     5D. X • X' = 0
```


EEL3701 Truth Table Proof of DeMorgan's Law (and use of Duality)

/(A + B) = /A • /B ⟺ (Dual) /(A • B) = /A + /B

A	B	/(A+B)	/A•/B	/(A•B)	/A + /B
0	0	1	1	1	1
0	1	0	0	1	1
1	0	0	0	1	1
1	1	0	0	0	0

QED

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An Example of DeMorgan's Law

Problem: Simplify /[(X + YZ) + W]

Solutions: Use DeMorgan's Law over and over

- Let **(X + YZ) = A** to make variables easier to work with

1. Substituting: /[(X + YZ) + W] = /[A+W]
   - >By *DeMorgan's Law,* **= /[A+W] = /A • /W**
2. But /A = /(X + YZ)
   - >By *DeMorgan's Law,* **= /(X + YZ) = /X • /(YZ)**
   - >By *DeMorgan's Law,* **= /(YZ) = /Y + /Z**
3. Putting 2 all together:
   - >/A = /(X + YZ) = /X • /(YZ) **= /X • (/Y + /Z)**
4. Substituting 3 into 1 ⇒
   - >Answer **= /X • (/Y + /Z) • /W**

(B+C)' = B'•C'
(B•C)' = B'+C'

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An Example of DeMorgan's Law

Problem: Simplify /[(X + YZ) + W]

Solutions: Use DeMorgan's Law over and over

4. Substituting 3 into 1 ⇒
   - >Answer **= /X • (/Y + /Z) • /W**
5. By the *distributive law,*
   - >Answer **= /X • /Y • /W + /X • /Z • /W**
6. Omitting dots for clarity,
   - >Answer **= /X /Y /W + /X /Z /W**

Preferred Answer = /W /X /Y + /W /X /Z

>Above is in **Lexical Order**

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EEL3701 Boolean Expressions in Lexical Order

- Consider XYZ=XZY=YXZ=YZX=ZXY=ZYX and X+Y+Z=X+Z+Y=Y+X+Z =Y+Z+X=Z+X+Y=Z+Y+X.
  - > Since the terms can be written in many ways (by associativity and/or commutativity), we need conventions.

Rule H1: You shall write all the Boolean expressions in **lexical** order, /A, A, /B, B,......., /Z, Z. Then each term is in **word** order.

- The answer of the previous example should, therefore, be:

  /W /X /Y + /W /X /Z

  rather than

  /X /Y /W + /X /Z /W or /Y /X /W + /Z /W /X

  (or many other possibilities)

EEL3701 Boolean Expressions in Lexical Order

- Consider $/D_2\,D_3\,D_0\,/D_1 = D_3\,/D_2\,/D_1\,D_0 = D_0\,/D_1\,/D_2\,D_3$
  - > Since the terms can be written in many ways (by associativity and/or commutativity), we need conventions.

Rule H2: You shall write all the Boolean expressions with same name in **reverse subscript numerical** order. Especially important within a single product (AND) term.

- For example, write the above product term as

  $\mathbf{D_3\,/D_2\,/D_1\,D_0}$

- For example, write

  $\mathbf{E\,A_{13}\,/A_{14} + \,/A_{15}}$ as $\mathbf{/A_{15} + /A_{14}\,A_{13}\,E}$

EEL3701 SOP (Sum of Products)

Definitions: Any function can be written as a ***sum of products*** that is as a disjunction of conjunctive terms (OR of ANDs). A ***minterm*** is a conjunctive (AND) term that is 1 in only 1 row of a truth table.

a	b	c	m_7	m_6	m_5	m_4	m_3	m_2	m_1	m_0	f(a,b,c)
0	0	0	0	0	0	0	0	0	0	1	1
0	0	1	0	0	0	0	0	0	1	0	0
0	1	0	0	0	0	0	0	1	0	0	1
0	1	1	0	0	0	0	1	0	0	0	1
1	0	0	0	0	0	1	0	0	0	0	0
1	0	1	0	0	1	0	0	0	0	0	1
1	1	0	0	1	0	0	0	0	0	0	1
1	1	1	1	0	0	0	0	0	0	0	0

$$f = \sum_i \text{Minterm}_i$$

f(a,b,c) = $m_0 + m_2 + m_3 + m_5 + m_6$

= /a/b/c + /a b /c + /a b c + a /b c + a b /c

Q: Can this be simplified?_____ Why?_____ How?______

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EEL3701 Minimum Sum Of Product (MSOP)

- When you cannot reduce an SOP further, this is called a minimum sum of products (MSOP) expression

Ex: Simplify the following expression (from previous page)

f(a,b,c) = $m_0 + m_2 + m_3 + m_5 + m_6$ = /a/b/c + /a b/c + /a b c + a/b c + a b/c

We quite often use X+/X = 1, X•1 = X, X+X = X, & X•X=X.

Thus, $m_0 + m_2$ = /a/b/c + /a b/c = /a /c (b + /b)= /a /c

$m_2 + m_3$ = /a b/c + /a b c = /a b (/c + c) = /a b

$m_2 + m_6$ = /a b/c + a b/c = (/a +a) b /c = b /c

f(a,b,c) = $m_0 + m_2 + m_3 + m_5 + m_6 = m_0 + m_2 + \mathbf{m_2} + \mathbf{m_2} + m_3 + m_5 + m_6$

= $(m_0 + m_2) + (\mathbf{m_2} + m_3) + (\mathbf{m_2} + m_6) + m_5$

= /a /c + /a b + b /c + a /b c

f(a,b,c) = /a b + /a /c + a /b c + b /c

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EEL3701 Algebraic Simplification - Boolean Algebra

Simplification using the axioms of Boolean Algebra

Ex: G_{SOP} = /A /B + A /B = (/A + A) /B = /B

/X + X = 1
X • 1 = X

Note: Truth table for /A /B + A /B is same as for /B

Let's view similar product of sums (POS) result

Ex: G_{POS} = (/A + /B)(A + /B)
= (/B + /A)(/B + A)

Commutative Law

G_{POS} = /B

Apply Distributive Law: (X+Y)(X+Z) = X+(YZ)
So (X+Y)(X+/Y) = X+(Y /Y) = X+0 = X
Now let X=/B, Y=/A ⇒ /Y = A

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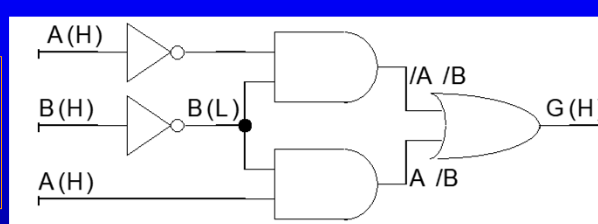
EEL3701 Algebraic Simplification for Simplified Circuits

- We want to simplify (and eliminate) literals (terms) because each literal is a gate input. The smaller the number of literals the better!

Ex: Any activation levels
G_{SOP} = /A /B + A /B



G_{SOP} = /B
Saved 4 gates!

G_{SOP} = /B
Wires are cheap!

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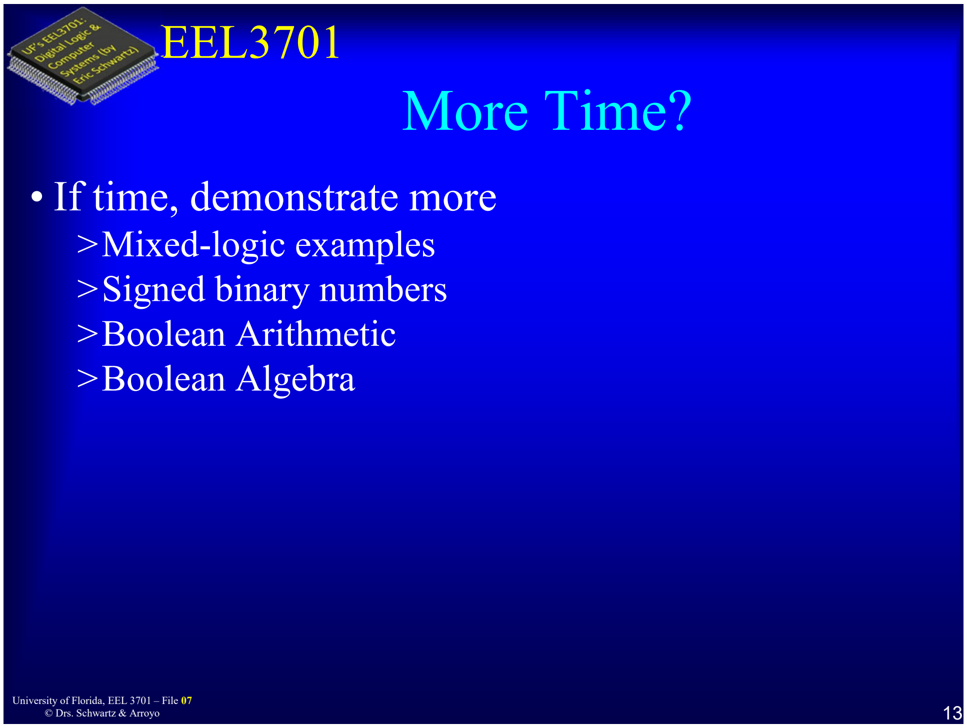
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More Time?
• If time, demonstrate more
>Mixed-logic examples
>Signed binary numbers
>Boolean Arithmetic
>Boolean Algebra
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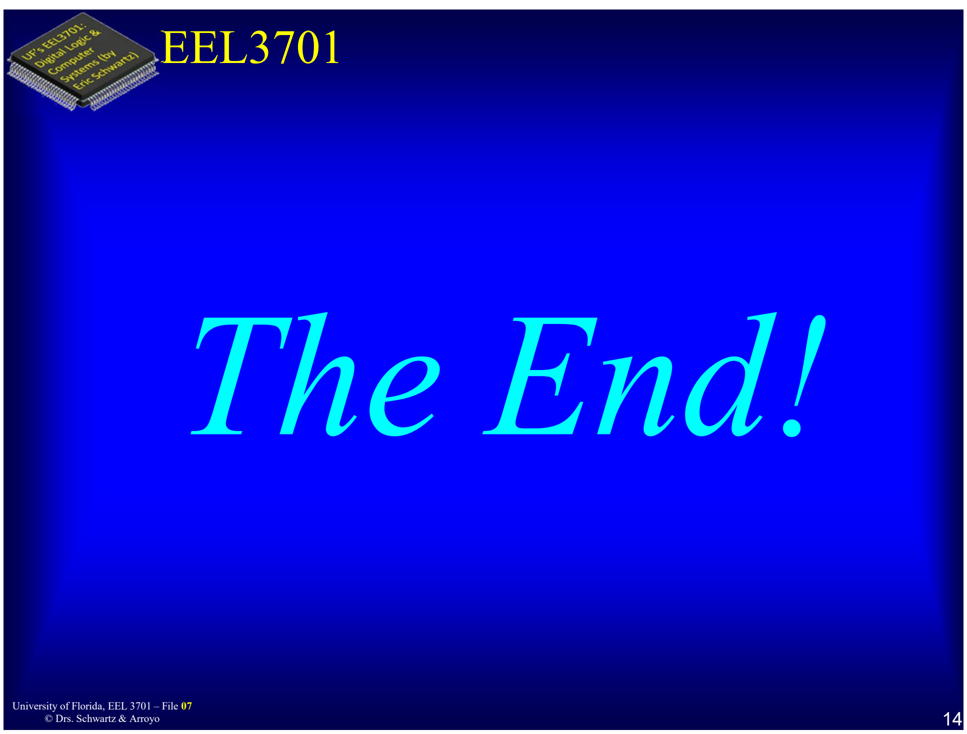
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The End!
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